

Two-body correlations in N=8 and 10 nuclei and effective neutron-neutron interactions in Tamm-Dancoff and two-particle RPA models.

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We apply a particle-particle RPA model to study the properties of the two-neutron valence wave function in nuclei ^{14}C , ^{12}Be , ^{11}Li and ^{14}Be . The RPA model takes account of two-body correlations in the cores so that it gives a better description of energies and amplitudes than models which assume a neutron closed shell (or subshell) core. With a Gogny neutron-neutron effective interaction or with the equivalent density dependent delta force we are able to reproduce the two-neutron separation energies in these nuclei and in the corresponding cores, except for ^9Li . These calculations suggest the same 2s-1p $_{1/2}$ shells inversion in ^{12}Be - ^{13}Be than in ^{11}Be .

I. INTRODUCTION

In an earlier work [1], we calculated the properties of the two-neutron valence pair in the nuclei ^{11}Li , ^{12}Be , ^{14}C using a Tamm-Dancoff model and assuming the core to have a closed 1p $_{3/2}$ neutron shell. Important ingredients in the calculations are the assumed pairing interaction and the role of the p $_{1/2}$ -s $_{1/2}$ shells inversion that is visible in the spectrum of ^{11}Be .

While the model gave a coherent and reasonable description of these nuclei it fails to describe ^6He (taking now an alpha-particle core) giving a two-neutron separation energy of several MeV instead of the experimental value of 0.97 MeV [2]. This has already been found by Esbensen et al. [3]. When one compares ^6He described as an alpha-particle + two neutrons to ^{14}C for example, one sees immediately an important difference between the two systems: for ^6He the core is well described in an Hartree-Fock model as a pure (1s) $^2_\nu$ (1s) $^2_\pi$ configuration [4] while we have long known [5] that in ^{14}C the core of ^{12}C is a mixture of states with neutrons in the 1p $_{3/2}$ or 1p $_{1/2}$ states, what is not taken into account in Tamm-Dancoff models.

In this work, we will include the core correlations using the two-particle RPA theory and also examine more broadly the dependence on the assumed effective two-neutron force. Zero range residual forces are very simple to use but are quite arbitrary and binding energies are not sufficient to fix uniquely the force. In a recent work Garrido et al. [6] have searched for a zero range density dependent force equivalent to the finite range Gogny interaction [7,8] (see also [9]). The authors fit the force in order to reproduce the gap calculated in nuclear matter with the Gogny force which has good pairing properties in finite nuclei. A fit to the whole domain of k_F values determines unambiguously the parameters of the force and tells what is the cut-off on neutron energy to be used. This last information is very important since a zero range interaction has no natural cut-off. The density independent part of the force reproduces the low energy properties of a free neutron-neutron system so that their force is equivalent to a realistic effective interaction in two-neutron and infinite systems. We will use this force in our problem and will discuss the results compared to the zero range pairing forces used in Tamm-Dancoff models.

In section II we briefly report on the results obtained in a pairing or Tamm-Dancoff model with three effective neutron-neutron interactions. In section III we recall the properties and equations of the particle-particle RPA model. The results of this model are presented and discussed in section IV for ^{14}C - ^{10}C , ^{12}Be - ^8Be and ^{11}Li - ^7Li and in section V for ^{14}Be - ^{10}Be with a discussion on the ^{13}Be and ^{11}Be spectra. At the end, section VI is devoted to our conclusions.

II. HAMILTONIAN AND EFFECTIVE INTERACTIONS

We first make a pairing, or Tamm-Dancoff, approximation to describe core + two neutron systems. Assuming an inert and closed sub-shell core for neutrons we diagonalise the two-neutron hamiltonian in a two-neutron subspace built on non occupied neutron states in the core:

$$H_{2n} = \frac{\mathbf{p}_1^2}{2m} + \frac{\mathbf{p}_2^2}{2m} + V_{nc}(1) + V_{nc}(2) + V_{nn}(1, 2) + \frac{(\mathbf{p}_1 + \mathbf{p}_2)^2}{2A_c m} \quad (1)$$

$$= h_{nc}(1) + h_{nc}(2) + V_{nn}(1, 2) + \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{A_c m} \quad (2)$$

where the one-neutron hamiltonian is:

$$h_{nc}(i) = \frac{\mathbf{p}_i^2}{2\mu} + V_{nc}(i) \quad (3)$$

$$V_{nc}(r) = -V_{NZ} \left(f(r) - 0.44r_0^2(1.s) \frac{1}{r} \frac{df(r)}{dr} \right) + 16a^2\alpha_n \left(\frac{df(r)}{dr} \right)^2 \quad (4)$$

$$f(r) = \left(1 + \exp\left(\frac{r-R_0}{a}\right) \right)^{-1} \quad (5)$$

$$V_{NZ} = U_0 - U_\tau \frac{N-Z}{A_c} \quad (6)$$

μ is the reduced mass equal to $\frac{A_c m}{A_c + 1}$; A_c , N , Z are respectively the mass, neutron and proton numbers in the core; $R_0 = r_0 A_c^{1/3}$ with $r_0=1.27$ fm, $a=0.75$ fm; the strengths U_0 and U_τ are taken the same as in our previous papers [1,10]. The last term of potential, eq.(4), simulates particle-phonon couplings contribution to the one-body potential [11]. The strengths α_n are fitted in each nucleus to reproduce the experimental $1p_{1/2}$, $2s$, $1d_{5/2}$ single neutron energies for ^{14}C and ^{12}Be and in order to get the measured two-neutron separation energy in ^{11}Li and ^{14}Be for $1p_{1/2}$ and $2s$ states which are not experimentally well known. Their numerical values can be found in refs.[1] and [10]. For higher neutron states the particle-phonon couplings are weak and we take $\alpha_n=0$. We use a discretisation of the continuum states with a radial box of 20 fm and orthonormalise the wave functions by the Schmidt method. In our previous papers the two-body term $\mathbf{p}_1 \cdot \mathbf{p}_2$ was neglected but as the approximation was the same for all nuclei: ^{14}C , ^{12}Be , ^{11}Li and ^{14}Be and the effective neutron-neutron interaction fitted to describe the properties of ^{14}C and ^{12}Be , then used in the other nuclei, the effect of such an approximation was minimised. We have checked that adding this term changes slightly the strength of the effective interaction (by few %) but gives the same agreements and predictions than previously.

We used in refs.[1,10] a zero range density dependent neutron-neutron interaction given by:

$$V_{nn}(1,2) = -V_0 \left(1 - x [\rho_c(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2})/\rho_0]^p \right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (7)$$

where ρ_c is the core density and $\rho_0=0.16\text{fm}^{-3}$. The above fit gives now $V_0=880 \text{ MeV}\cdot\text{fm}^3$, $x=0.93$ and $p=1.2$ close to what is used in the literature [9,12]. With the same force we have calculated ^6He as an alpha-particle plus two neutrons. Taking V_{nc} as the neutron-alpha particle potential fitted by Satchler et al. [13] to low energy $l=1$ phase shifts we get a two-neutron separation energy of about 4 MeV in ^6He while it is 0.97 MeV experimentally [2]. Similar strong binding was found by Esbensen et al. [3] who modify the n-n interaction in order to get the experimental value. However this is not satisfactory for us because we always require that our effective interaction leads to an overall agreement for all systems since the density dependent term should take account of the dependence of the effective interaction upon the nucleus. We have looked for different possible sources of such unexpected disagreement: choice of neutron-alpha particle interaction, discretisation of the continuum which will be discussed in a forthcoming paper and effect of the cut-off on neutron energy. However none of these effects is responsible for such a bad result.

The force of eq.(7) has three parameters which are not independently determined from a fit of energy spectra. In a recent paper Garrido et al. [6] have fitted the three parameters of V_{nn} in order to get the same gap, $\Delta(k_F)$, in nuclear matter than obtained with a Gogny finite range effective interaction [7,8]. They have shown that to get agreement over all the domain of k_F they have to take $p=0.47$, $x=0.45$ with a cut-off, ϵ_c^0 , on the neutron energy of 50-60 MeV. The cut-off energy determines the strength V_0 if one assumes that the density independent part of the interaction should reproduce the properties of a free neutron-neutron system. It gives the following relation between V_0 , ϵ_c^0 and the neutron-neutron scattering length a_{nn} [3] :

$$V_0 = 2\pi^2 \frac{\hbar^2}{m} \frac{1}{k_c^{(0)} - \frac{\pi}{2a_{nn}}} \quad (8)$$

$$k_c^{(0)2} = \frac{2m\epsilon_c^0}{\hbar^2} \quad (9)$$

a_{nn} is experimentally -18.5 fm then very large. If we replace it by $-\infty$ and take $\epsilon_c^0=60$ MeV this relation gives $V_0 = 480 \text{ MeV}\cdot\text{fm}^3$ as used in ref.[6].

For free neutrons, the neutron energy is only kinetic energy while in finite nuclei the neutrons are in the potential V_{nc} . Therefore when using eqs.(8-9) the cut-off energy has to be calculated from the bottom of the single-particle well and ϵ_c^0 replaced by :

$$\epsilon_c^0 = \epsilon_c + V_{NZ} \quad (10)$$

where V_{NZ} is the depth of our one-body potential, eq.(6), and ϵ_c the cut-off energy for a neutron described by the hamiltonian of eq.(3). In our calculations we take neutron states up to an energy $\epsilon_c \simeq 10$ MeV for all nuclei. V_{NZ} depends on proton and neutron numbers in the core and varies between about 40 MeV for ^{11}Li and 50 MeV for ^{14}C , giving an equivalent cut-off in free two-neutron system or nuclear matter of 50-60 MeV as required by the fit of ref.[6]. Note that taking ϵ_c the same for all nuclei implies that V_0 will be slightly different for ^{11}Li , ^{12}Be and ^{14}C .

This new zero range effective interaction has very different parameters compared to the usual ones. We have made the same calculations as previously with this force for our systems ^{14}C , ^{12}Be and ^6He . ^{11}Li results are not reported in the table but with neutron energies close to experimental values it is not bound. The results are reported in Table I where they are compared to the measured two-neutron separation energy, S_{2n} [2]. We see two interesting facts: the calculated S_{2n} in ^6He is now 1.02 MeV close to the experimental value but it is much too low in the other nuclei. Since this zero range force is constructed to reproduce the Gogny force in nuclear matter and in a free two-neutron system we thought that this equivalence could fail in our finite nuclei which are quite far from both systems.

Then we have made the same Tamm-Dancoff calculation with the genuine Gogny force, D1 or D1S [7,8]. The Gogny forces are the sum of central density dependent and spin-orbit terms. For two neutrons coupled to 0^+ the spin-orbit term gives very small contribution to pairing matrix elements and can be neglected [14]. Therefore we have made our calculations, keeping the central term only. The results are very close for the two forces and we present them for the force D1S only. The results with the Gogny force are presented in Table I.

Force	^{14}C	^{12}Be	^6He
Gogny	11.8	2.23	0.94
δ force	11.7	1.95	1.02
exp.	13.12	3.67	0.97

TABLE I. Two-neutron separation energies in MeV calculated in Tamm-Dancoff model with the Gogny DS1 force and a zero range force of eq.(7) with $V_0=480$ MeV.fm³, $p=0.47$ and $x=0.45$

We see that they are the same as obtained with its zero range substitute what tells us that:

1- the equivalence between the two forces, with zero or finite range, shown for a free system and nuclear matter holds in all considered finite nuclei. We give the results for $V_0=480$ MeV.fm³, the same for all nuclei, but as already mentioned V_0 should be larger in ^{12}Be than in ^{14}C following relations (6), (8) and (10) what will improve the agreement between the two series of results.

2- the range of the effective force is not responsible for the inability of the nuclear model to describe simultaneously ^6He and the p-shell nuclei.

The agreement for ^6He on one side, the disagreement for the other nuclei on the other side seem at first sight a contradiction of the model. However it can be understood when we go back to the first assumption of the Tamm-Dancoff model that the core is inert with neutrons filling the lowest shells. For ^6He the core, an alpha-particle, is strongly bound and very well described in an Hartree-Fock model with neutrons and protons filling the 1s shell [4]. It means that the assumption of a closed shell core (implicitly assumed when we put our extra- neutrons on the unoccupied shells with probability one) is valid in this case. However for the other nuclei we know that the cores are not good closed shell nuclei. It is known for long in the case of ^{12}C from the work of Cohen-Kurath [5]. For ^{12}Be shell model calculations also show a large deviation from a pure state as it is required by experiments [15–18]. It is also obvious in our calculations for ^{12}Be [1] or in three-body Fadeev model calculations of Thompson et al. [19] where ^{12}Be is described as a core of ^{10}Be plus two neutrons and found as a large mixture of $(2s)^2$, $(1p_{1/2})^2$ and $(1d_{5/2})^2$ two-neutron components while it is considered as a pure $(2s)^2$ or $(1p_{1/2})^2$ state when one studies ^{14}Be .

A model which takes account of two-body correlations in the ground state of the core is the particle-particle RPA model more generally called pair vibration model [20–22]. We briefly recall what this model is for two neutrons outside a core which, in an independent particle model, would have closed shells (or subshells) for neutrons.

III. PARTICLE-PARTICLE RPA MODEL

Particle-particle RPA relies on the expansion of the two-particle (two-hole) Green's function in terms of ladder diagrams with upward and backward going diagrams as shown in Fig.1. Note that a summation over upward going diagrams only leads to the Tamm-Dancoff approximation of the previous section. From the related

approximate integral equation satisfied by this RPA two-particle Green's function, a system of equations is derived which describes simultaneously the A_c+2 and A_c-2 systems where A_c characterises the core which in an independent particle model would be described as a closed shell (or sub-shell) nucleus.

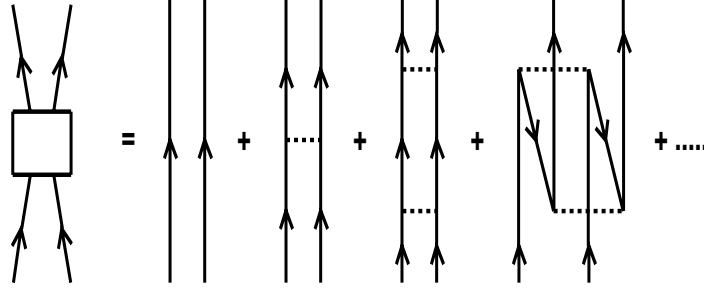


FIG.1: RPA diagrammatic expansion of the two-particle Green's function.

We here apply this method to our problem of two neutrons outside a core with N_c neutrons assuming that the protons are not disturbed when one adds or subtracts two neutrons.

Let's define $a=(a_1,a_2)$, $b=(b_1,b_2)$... two-neutron configurations with the neutrons in states a_1,a_2 ... unoccupied in the Hartree Fock core ground state, α,β ... two-neutron configurations with the neutrons in occupied states α_1,α_2 ... and two-neutron amplitudes or spectroscopic factors as:

$$X_\alpha(J, M) = \langle N_c + 2 | A_\alpha^+(J, M) | N_c, 0 \rangle \quad (11)$$

$$X_a(J, M) = \langle N_c + 2 | A_a^+(J, M) | N_c, 0 \rangle \quad (12)$$

where the pair operators are given by:

$$A_a^+(J, M) = \sum_{m_{a_1}, m_{a_2}} (j_{a_1}, j_{a_2}, m_{a_1}, m_{a_2} | J, M) a_{a_1}^+ a_{a_2}^+ \quad \text{with } a_1 \leq a_2 \quad (13)$$

$$A_\alpha^+(J, M) = \sum_{m_{\alpha_1}, m_{\alpha_2}} (j_{\alpha_1}, j_{\alpha_2}, m_{\alpha_1}, m_{\alpha_2} | J, M) a_{\alpha_1}^+ a_{\alpha_2}^+ \quad \text{with } \alpha_1 \leq \alpha_2 \quad (14)$$

a_i^+ is the creation operator of a neutron in state j_i, m_i , $|N_c + 2\rangle$ and $|N_c, 0\rangle$ are respectively the RPA wave functions of the $N_c + 2$ nucleus in excited or ground state and of the core in its ground state. We see immediately that the amplitudes X_α are non zero only if the core ground state has 2p-2h, 4p-4h, ... components. Assuming that all X_α are zero, one gets back to the Tamm-Dancoff approximation.

In the same way we define two-neutron hole amplitudes for the $N_c - 2$ nuclei from A_a and A_α , the annihilation operators for a pair which are hermitian conjugates of A_a^+ and A_α^+ defined by eqs.(11-12). They are:

$$Y_a(J, M) = \langle N_c - 2 | A_a(J, M) | N_c, 0 \rangle \quad (15)$$

$$Y_\alpha(J, M) = \langle N_c - 2 | A_\alpha(J, M) | N_c, 0 \rangle \quad (16)$$

For the N_c-2 nuclei Y_a are non zero only if the core ground state is not a pure HF state but has 2p-2h, 4p-4h, ... components. Then these anomalous components X_α and Y_a give a measure of the two-body correlations in the cores. For a given spin and parity the RPA amplitudes X and Y satisfy the same system of equations:

$$(E - \epsilon_a)x_a - \sum_b \langle a | V_{nn} | b \rangle x_b - \sum_\beta \langle a | V_{nn} | \beta \rangle x_\beta = 0 \quad (17)$$

$$(E - \epsilon_\alpha)x_\alpha + \sum_b \langle \alpha | V_{nn} | b \rangle x_b + \sum_\beta \langle \alpha | V_{nn} | \beta \rangle x_\beta = 0 \quad (18)$$

where x are the amplitudes X or Y as explained below. The two-body matrix elements are antisymmetrised and $\epsilon_a, \epsilon_\alpha$ the sum of unperturbed energies of the two neutrons in configurations a, α respectively. The one-neutron states are described by the one-body effective potential of eq.(4) what means that we expand our two-body Green's function of Fig.1 in terms of one-body propagators which include particle-phonon couplings as given in Fig.2.

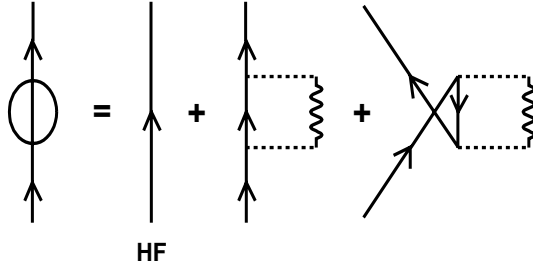


FIGURE 2. One-particle Green's function with inclusion of particle-phonon coupling diagrams.

If we take N configurations a, b, \dots and M configurations α, β, \dots the equations (17,18) have $N+M$ solutions. N solutions correspond to the $N_c + 2$ nucleus with eigenvalues E_{nJ} and amplitudes $X^{(n)}$ where n labels the state such that:

$$E_{nJ} = E_{nJ}(N_c + 2) - E_0(N_c) \quad (19)$$

$$X_a^{(n)}(J) = x_a^{(n)}, \quad X_\alpha^{(n)}(J) = x_\alpha^{(n)} \quad (20)$$

the amplitudes satisfy the following orthonormalisation relation:

$$\sum_a X_a^{(n)} X_a^{(n')} - \sum_\alpha X_\alpha^{(n)} X_\alpha^{(n')} = \delta_{nn'} \quad (21)$$

The M other solutions (labelled by the index m) are eigenstates of the $N_c - 2$ nucleus with eigenvalues E_{mJ} and amplitudes Y given by:

$$\mathcal{E}_{mJ} = (E_{mJ}(N_c - 2) - E_0(N_c)) = -E_{mJ} \quad (22)$$

$$Y_a^{(m)}(J) = x_a^{(m)}, \quad Y_\alpha^{(m)}(J) = x_\alpha^{(m)} \quad (23)$$

with the orthonormalisation condition:

$$\sum_a Y_a^{(m)} Y_a^{(m')} - \sum_\alpha Y_\alpha^{(m)} Y_\alpha^{(m')} = -\delta_{mm'} \quad (24)$$

We see from eqs.(19) and (22) that the lowest energies E_{00} and \mathcal{E}_{00} give the two-neutron separation energy in the $N_c + 2$ and N_c nuclei respectively with the relations:

$$S_{2n}(N_c + 2) = -E_{00} \quad (25)$$

$$S_{2n}(N_c) = \mathcal{E}_{00} \quad (26)$$

Then, contrary to particle-hole RPA, all solutions of eqs.(17,18) correspond to physical states. The separation between N and M states follows from the relative importance of x_a and x_α amplitudes: for the $N_c + 2$ nucleus the amplitudes x_a are larger than x_α and inversely for $N_c - 2$ nucleus.

This model is now applied to the p-shell nuclei. As noticed already ${}^6\text{He}$ described as an α particle plus two neutrons is very likely well reproduced by a Tamm-Dancoff model since the core is a good Hartree-Fock system. On the other hand, the $N_c - 2$ nucleus (di-proton) is not a bound system and the RPA model is not reliable in this case so that we discuss the p-shell nuclei only.

IV. RESULTS FOR $N=8$ NUCLEI: ${}^{14}\text{C}$, ${}^{12}\text{Be}$ AND ${}^{11}\text{Li}$

We make the calculation for the two effective interactions, the Gogny finite range interaction and the zero range interaction of eq.(7) with the parameters p and x fitted by Garrido et al. and a strength V_0 fitted to give the same RPA energies as the Gogny force. Afterwards we shall compare with the strength given by eqs.(8-10). The two-neutron subspace (a, b, \dots) is the same as in section II with a normal ordering of $1p_{1/2}$ - $2s$ shells in C-isotopes but an inversion of these two shells in Be and Li. For the occupied neutron states (α, β, \dots) we take the two shells $1s_{1/2}$ and $1p_{3/2}$. The $1p_{3/2}$ neutron energy is taken as the known one-neutron separation energy for the corresponding core (${}^{12}\text{C}$ or ${}^{10}\text{Be}$ or ${}^9\text{Li}$) [2]. For the $1s$ shell we have no experimental information and

we take it as a parameter by changing the depth of our Saxon-Woods potential. This very deep state has very little effect on the N_c+2 states but a non negligible effect on the energy of the N_c-2 ground states. Then we take the 1s energy in order to get close agreement, if possible, with the experimental two-neutron separation energy in the core nucleus. This procedure gives $\epsilon(1s)=-32, -32$ and -28 for ^{12}C , ^{10}Be and ^9Li respectively.

	^{14}C	^{12}C	^{12}Be	^{10}Be	^{11}Li	^9Li
Gogny	12.9	32.	3.69	8.2	0.37	3.8
δ force	12.9	32.5	3.6	8.52	0.34	3.76
exp.	13.12	31.8	3.67	8.48	0.34 ± 0.05	6.1

TABLE II. Same as Table I for RPA model and $V_0=500, 510$ and 560 MeV.fm^3 for ^{14}C - ^{12}C , ^{12}Be - ^{10}Be and ^{11}Li - ^9Li respectively

In Table II are presented S_{2n} , the two-neutron separation energies in N_c+2 (^{14}C , ^{12}Be and ^{11}Li) and N_c (^{12}C , ^{10}Be and ^9Li) nuclei given by the lowest eigenvalues E_{00} and \mathcal{E}_{00} respectively, following relations (25) and (26). The results are given for the Gogny force and the equivalent zero range force. In Table III we give the amplitudes X_a and X_α for $N_c + 2$ nuclei and Y_a and Y_α for $N_c - 2$ nuclei corresponding to the ground states and obtained with the Gogny interaction. The zero range interaction gives nearly identical eigenvectors.

For ^{14}C - ^{12}C and ^{12}Be - ^{10}Be , S_{2n} is close to the experimental values while we have no free parameters, apart from the 1s-energy which turns out from the fitting procedure to be very close to a typical Hartree-Fock energy. We see that the amplitudes X_α for ^{14}C and ^{12}Be are for $\alpha = (1p_{3/2})^2$ rather large and larger in ^{12}Be than in ^{14}C . This large value means that in the ground state of the cores, ^{12}C and ^{10}Be , there are large components of 2nh-2np states with at least two holes in the $1p_{3/2}$ shell. This is in complete agreement with the shell model results for ^{12}C [5] and for ^{12}Be [15–18]. Furthermore we see that, if in ^{14}C the amplitude for the two neutrons in a $1p_{1/2}$ state is very large, in ^{12}Be we have comparable amplitudes for $(2s)^2$, $(1p_{1/2})^2$ and $(1d_{5/2})^2$ configurations. This means that, together with the fact that $X_\alpha = 0.46$, ^{12}Be cannot reasonably be considered as a closed shell nucleus as done in many papers on ^{14}Be .

Our RPA equations give simultaneously the amplitudes Y_a and Y_α of eqs.(15-16) for ^{10}C and ^8Be . Here the Y_a are the anomalous amplitudes reflecting again correlations in the cores. We see consistency with the results for ^{14}C and ^{12}Be . The amplitudes Y_a are larger in ^8Be than in ^{10}C with a distribution over several two-neutron states revealing a very complicated structure of ^8Be .

The ^{11}Li - ^7Li systems present more ambiguity than the previous ones. Now we know from break-up experiments that the lowest neutron resonance in ^{10}Li is an s-state at 0.1-0.2 MeV [23] as required by several calculations [1,24,25] and that the next state is a $p_{1/2}$ resonance at $0.54 \pm 0.06 \text{ MeV}$ [26].

	$X_a(Y_a)$				$X_\alpha(Y_\alpha)$	
	$(2s)^2$	$(1p_{1/2})^2$	$(1p_{1/2}, 2p_{1/2})$	$(1d_{5/2})^2$	$(1p_{3/2})^2$	$(1s)^2$
^{14}C	-0.12	0.96	-0.05	-0.28	0.19	-0.09
^{10}C	-0.05	0.15	-0.02	-0.14	0.98	-0.32
^{12}Be	-0.48	0.76	-0.29	-0.44	0.60	-0.09
^8Be	-0.16	0.34	-0.17	-0.30	1.18	-0.12
^{11}Li	0.66	-0.56	0.48	0.04	-0.45	0.07
^7Li	-0.12	0.22	-0.27	0.02	1.12	-0.11

TABLE III. The most important RPA amplitudes (X_a, X_α for $N_c + 2$ nuclei, Y_a, Y_α for $N_c - 2$ nuclei)

The results, presented in Tables II and III, have been obtained with $\epsilon(1s) = -28 \text{ MeV}$, $\epsilon(2s) = 0.19 \text{ MeV}$ and $\epsilon(p_{1/2}) = 0.6 \text{ MeV}$ close to measurements for the two last ones and reasonable for the first one. We first see that S_{2n} in ^{11}Li is in good agreement with the experimental value, $0.34 \pm 0.05 \text{ MeV}$ [27]. The anomalous amplitude X_α for $\alpha=(1p_{3/2})^2$ is as large as it was in ^{12}Be revealing strong deviation from closed shell in ^9Li . We also see that the amplitudes for $(1p_{1/2})^2$ and $(2s)^2$ configurations are similar, as required by the analysis of break-up experiments [28]. However the S_{2n} value for ^9Li is very far from the measured value, 3.85 instead of 6.1 MeV. This disagreement, found only in Li, cannot be improved by a reasonable change of $\epsilon(1s)$ which is the only parameter of the calculation. It could come from the single last proton which is on the same shell as the active neutrons in ^7Li and ^9Li but we did not find any simple way to evaluate such an effect.

In the case of Li isotopes, there is an ambiguity coming from higher neutron states. Experimentally one has not seen the $d_{5/2}$ resonance, therefore we have calculated it with the bare Woods-Saxon potential even though we know that in ^{11}Be one has to add a surface term which lowers the $d_{5/2}$ energy. However the effect of such

a term should not affect the results since, as seen in Table III, the amplitude for the $(1d_{5/2})^2$ configuration is very small. Also, because ^{10}Li is unbound, all neutron states are in the continuum with the result that continuum non-resonant states play a more important role in the determination of the ^{11}Li energy. This effect is particularly important for the discretised $2p_{1/2}$ state which is taken as an eigenstate of the bare Woods-Saxon potential. Because the modification of the potential by particle-vibration couplings is very large for the $1p_{1/2}$ resonance, this $2p_{1/2}$ state comes at an energy close to the $1p_{1/2}$, giving a large overlap between the two states. We know that particle-vibration couplings are more important for states close to the Fermi surface and can be neglected for higher states in usual nuclei. However in ^{10}Li these couplings are very strong and very likely still efficient for the $2p_{1/2}$ state. Then as a test we have made the calculation with the same surface contribution to the one body potential for $1p_{1/2}$ and $2p_{1/2}$ states, taking the bare potential for the other p-states which anyway have much higher energies and therefore have a weaker effect on the ground state energy and wave function. The amplitude X_a for $a=(1p_{1/2}, 2p_{1/2})$ is weaker, S_{2n} slightly smaller. To recover the same value one has to take $\epsilon(1p_{1/2})=0.53$ MeV, then at the experimental energy, but these differences are not significant.

Going back to Table II we see that the strengths V_0 of the zero range interaction fitted to recover the same energy than the Gogny interaction are 500, 510 and 560 MeV.fm³ for ^{14}C , ^{12}Be and ^{11}Li respectively. They follow closely the dependence on V_{NZ} given by eqs.(8-10) with a value for ^{14}C very close to 480 MeV.fm³ used in ref.[6]. We again find the same equivalence between the two forces, with zero or finite range, as derived for free neutrons and nuclear matter. It seems from our study that the need of a much stronger zero range force to get experimental binding energies in simple pairing models, as reminded in Section II, is in fact a way to take implicitly account of two-body correlations in the core which are neglected in the model.

V. RESULTS FOR ^{14}Be (N=10).

The problem of ^{14}Be is not yet completely clarified, both experimentally and theoretically. Experimentally one knows that ^{13}Be has a $d_{5/2}$ resonance at 2.01 MeV above the $n+^{12}\text{Be}$ threshold [29], a lower resonance at about 0.8 MeV seen in the reaction $^{14}\text{C}(^{11}\text{B}, ^{12}\text{N})^{13}\text{Be}$ [30] which has no spin or parity assignment and, from a recent experiment [31] using fragmentation of ^{18}O and detecting neutrons in coincidence with ^{12}Be , a $1/2^+$ resonance below 0.2 MeV which should be the ground state of this unbound nucleus. Theoretical models describing ^{14}Be as two neutrons outside a core of ^{12}Be , where the neutrons fill the $1s$, $1p_{3/2}$ and $1p_{1/2}$ shells, have either to lower the $d_{5/2}$ resonance or to assume a bound $2s$ state therefore to bind ^{13}Be to get a correct binding energy in ^{14}Be [19]. In a first paper [10] based on a two-neutron Tamm-Dancoff model and with a zero range effective interaction fitted on ^{14}C and ^{12}Be we found that an inversion of the $1p_{1/2}$ - $2s$ shells leads to the correct binding energy in ^{14}Be without modifying the known $d_{5/2}$ energy nor assuming a bound ^{13}Be . This calculation, as others, assumes a closed shell nucleus of ^{12}Be where the neutrons fill the $1s$ - $1p_{3/2}$ - $2s$ shells, while we have seen above that the ground state of ^{12}Be has amplitudes X_a over several two-neutron configurations and, because of large X_a , has very likely components on more complicated configurations. The particle-particle RPA applied to ^{14}Be takes this into account and will provide a description of the core consistent with the RPA amplitudes of Table III.

We have made the RPA calculation assuming a normal ordering of shells: $1s$, $1p_{3/2}$, $1p_{1/2}$, $2s$,... with a Hartree Fock state where the p-shell is filled for neutrons. With $\epsilon(1p_{1/2})=-3.15$ MeV, given by the neutron separation energy in ^{12}Be [2], a $1d_{5/2}$ state at 2 MeV, the experimental value, and a $2s$ state at 7 keV, very close to the threshold, we get $S_{2n}=0.7$ MeV much too low compared to the experimental value of 1.34 ± 0.11 MeV [2]. For ^{12}Be we get $S_{2n}=3.26$ MeV, also too small. Moreover the amplitudes X_a are small indicating weak correlations in ^{12}Be in disagreement with the results of the previous section. Therefore we find again that we are not able to describe satisfactorily ^{14}Be when we assume a normal ordering of shells in ^{12}Be .

We now assume an inversion of the two shells $2s$ and $1p_{1/2}$ as in ^{11}Be . The configurations α are built on $1s, 1p_{3/2}$ and $2s$ states while the configurations a include neutrons in $1p_{1/2}$ state. The results are summarised in Table IV where we give S_{2n} for ^{14}Be and the RPA amplitudes for ^{14}Be and ^{10}Be . They are obtained for a Gogny effective interaction with the $d_{5/2}$ resonance at the experimental energy of 2 MeV, a $1p_{1/2}$ resonance at 0.68 MeV, an occupied $2s$ shell with an energy of -3.15 MeV given by the experimental neutron separation energy in ^{12}Be and a $1p_{3/2}$ state at -5.6 MeV. For the α configurations we have taken the $(1p_{3/2})^2$, $(1s, 2s)$ and $(2s)^2$ states. We have checked that adding the $(1s)^2$ states does not change our results.

We get 3.65 MeV for the two-neutron separation energy in ^{12}Be . By comparing with the result of Table II we see that the values of S_{2n} in ^{12}Be found in both calculations are very close showing the coherence of the model. Indeed we get 3.65 MeV when ^{12}Be is considered as the core for the ^{14}Be - ^{10}Be systems while it is 3.69 MeV when it is described as two neutrons outside a core of ^{10}Be .

The $p_{3/2}$ energy cannot be deduced directly from experiments. However ^{11}Be has a $3/2^-$ state at 3.9 MeV excitation energy [32], then at 7 MeV when referred to the ^{12}Be core, with a very small width and has certainly a large component of one neutron-hole in ^{12}Be mixed with $2h$ - $1p$ components which have higher energies.

Therefore a hole energy of 5.6 MeV is not unrealistic.

According to our discussion for ^{11}Li we have done the calculation using the same surface potential for $1p_{1/2}$ and $2p_{1/2}$ states. We have then to lower the $1p_{1/2}$ energy to 0.56 MeV if we leave all other states to be the same, what does not modify qualitatively our conclusions.

	$S_{2n}(\text{MeV})$	$(1p_{1/2})^2$	$(1p_{1/2}, 2p_{1/2})$	$(1d_{5/2})^2$	$(1p_{3/2})^2$	$(2s)^2$
^{14}Be	1.30	-0.46	0.62	0.74	-0.64	0.46
^{10}Be	-	-0.20	0.31	0.46	-0.73	1.

TABLE IV. RPA energy in MeV for ^{14}Be and main amplitudes for ^{14}Be and ^{10}Be obtained with the D1S Gogny force.

The amplitudes given in Table IV show again strong correlations in ^{12}Be . Indeed the anomalous amplitudes in ^{14}Be are 0.64 and 0.46 for $(p_{3/2})^2$ and $(2s)^2$ respectively what means that in ^{12}Be components on configurations with two holes on these shells are important. Note that the components with two holes in the 2s-shell are qualitatively consistent with the amplitudes X_a of Table III for $a=(1p_{1/2})^2$ and $(1d_{5/2})^2$. However RPA gives only overlap of wave functions and a strict and direct comparison between amplitudes derived in the two calculations is not possible. To make a direct comparison one has to calculate wave functions what can be made using a quasi-boson approximation as was done in the past for particle-hole RPA correlations.

Our fitted value $\epsilon(1p_{1/2})=0.68$ MeV is consistent with an unbound $1/2^-$ state in ^{13}Be which would be at 0.68 MeV above the $^{12}\text{Be} + n$ threshold, close to the experimental resonance at 0.8 MeV. However the recent experiment using fragmentation of ^{18}O [31] shows a low energy (≤ 0.2 MeV) s-wave strength in ^{13}Be what, in an independent particle model, would mean that the lowest unoccupied shell in ^{12}Be is an s shell and would reject the possibility of inversion. In our model however ^{13}Be is described as a neutron added to the correlated core of ^{12}Be which is a mixture of many different states, in particular it has large components on configurations with a closed $1p_{3/2}$ shell plus two neutrons on the 2s or on the $1p_{1/2}$ shell. Therefore these two components will give in ^{13}Be two different states, a $1/2^-$ state built on the first one with the last neutron on the $p_{1/2}$ shell (the 2s-shell is filled) and a $1/2^+$ state built on the second one with the last neutron on the empty 2s shell. In a weak coupling model, because of the known inversion in $n+^{10}\text{Be}$ system, one can show that the $1/2^+$ state is lower than the $1/2^-$ state by about 0.32 MeV and at about 0.3 MeV above the $^{12}\text{Be}+n$ threshold what is in qualitative agreement with the experimental state seen recently. Consequently a $1/2^+$ ground state in ^{13}Be is not in contradiction with an inversion of the 2s- $1p_{1/2}$ shells.

There are several other arguments in favor of this inversion in ^{12}Be - ^{13}Be . The first one relies on the recent measurement of the $B(E2)$ for the 2^+ state at 2.1 MeV in ^{12}Be which is found to be the same as in ^{10}Be for the 2^+ state at 3.4 MeV [33]. Because the inversion in ^{11}Be is related to the large value of the $B(E2)$ in ^{10}Be [11,34] there is no reason why the effect should be smaller in ^{13}Be . Moreover the phonon has a smaller energy in ^{12}Be than in ^{10}Be what is expected to give an enhancement of the coupling [11]. Note that this large $B(E2)$ was predicted in ref.[10]. Further arguments are found by looking at the ^{11}Be spectrum. Indeed if ^{13}Be can be described as a neutron added to a core of ^{12}Be , ^{11}Be can be described as a hole in the same core of ^{12}Be . Therefore the $1/2^+$ ground state of ^{11}Be is expected to correspond mainly to a hole in the last occupied shell in the Hartree-Fock ground state of ^{12}Be what implies that this shell should be an s shell, not a $p_{1/2}$. The first excited states can be obtained as two holes coupled to 0^+ plus a neutron on the $p_{1/2}$ or the $d_{5/2}$ shell therefore will be a $1/2^-$ and a higher $5/2^+$ states respectively, in agreement with the experimental spectrum. If we assume that the residual interaction between the two holes and the particle are similar if the particle is on a $p_{1/2}$ or $d_{5/2}$ shell we find a difference between the excitation energies of the two states of 1.32 MeV while experimentally it is 1.45 MeV. One sees that the inversion is able to give a coherent description of ^{11}Be , ^{13}Be and ^{14}Be .

Because any model relies on approximations one should be aware of the difficulty to draw a precise scheme. For this reason it is desirable to compare results of different models. Our results without inversion of shells agree with those of Thompson and Zukhov [19]. We now make a comparison with the work of Descouvemont [35,36]. He has calculated in the generator coordinate model (GCM) ^{13}Be and ^{14}Be as $^{12}\text{Be} + n$ and $^{12}\text{Be} + n+n$ systems respectively. The core of ^{12}Be is described by a filled 1p-shell for the neutrons while the two protons of the 1p-shell can couple to different states, ground and excited 0^+ , 1^+ and 2^+ states. Qualitatively it is equivalent to our Tamm-Dancoff approach where contribution of core excited states are put in our one-body neutron potential and where the neutrons of the core are assumed to fill the 1p-shell. The GCM calculations lead to a slightly bound $1/2^+$ ground state for ^{13}Be and a ^{14}Be bound by 1.1 MeV. In our Tamm-Dancoff approach with the zero range force fitted on ^{14}C and ^{12}Be , a 2s neutron state at -90 keV and a filled neutron 1p-shell (therefore without inversion) we get for ^{14}Be a binding energy of -0.93 MeV what is close to the GCM result. The three models, three-body Faddeev, GCM or simple pairing, lead to similar results but are not able

to give rise to good agreement for ^{13}Be and ^{14}Be when in ^{12}Be the neutrons are assumed to fill the $1p_{3/2}-1p_{1/2}$ shells.

A different work [37] using a density dependent relativistic mean field model calculates $^{12}\text{Be}-^{14}\text{Be}$ assuming closed $1p$ and $1p-2s$ neutron shells respectively. It gives too large one- neutron and two-neutron separation energies in both systems. This result, together with the assumption of closed shells in both ^{12}Be and ^{14}Be , is not in agreement with other calculations.

VI. CONCLUSIONS

We have first shown that a two-neutron Tamm-Dancoff model with a zero range density dependent neutron-neutron interaction fitted on ^{14}C and ^{12}Be gives simultaneously good results for ^{11}Li and ^{14}Be but fails to describe ^6He . The zero range force necessary to get agreement in C-Be-Li nuclei has very different parameters compared to the parameters fitted by Garrido et al. to reproduce the gap calculated in nuclear matter with the finite range Gogny effective interaction. The same Tamm-Dancoff calculation with these two forces shows that they are still equivalent in our finite nuclei and gives a good binding energy in ^6He but too weak binding in $N=8$ nuclei. This result is well understood in terms of two-body correlations in the cores. Indeed we know that the alpha-particle is well described in Hartree-Fock model. Then two-body correlations in ^4He are unefficient while we know for long from shell model calculations that the cores of ^{12}C and ^{12}Be and very likely ^9Li are not pure closed shell nuclei as assumed in a pairing model. It is also obvious for ^{12}Be in our calculation: when it is described as a core of ^{10}Be plus two neutrons its wave function is a mixture of $(2s)^2$, $(1p_{1/2})^2$ and $(1d_{5/2})^2$ two-neutron states while in the study of ^{14}Be it is considered as a pure $(2s)^2$ or $(1p_{1/2})^2$ state what yields inconsistency of the model.

The particle-particle RPA model is well adapted to take into account such correlations and indeed the model applied to $^{14}\text{C}-^{11}\text{Li}-^{12}\text{Be}-^{14}\text{Be}$ gives now with those realistic forces very good agreement with experimental binding energies. It gives also the two-neutron separation energy in the cores. For $^{12}\text{C}-^{10}\text{Be}$ and ^{12}Be (the latter being considered as a core in the calculation of ^{14}Be) the agreement with measurements is also very good. However it is too small in ^7Li very likely due to the single proton in the $p_{3/2}$ shell. The model gives also two-neutron and two-neutron hole amplitudes in the wave functions (spectroscopic factors) which are related to the amount of two-body correlations introduced in the cores. This effect is always large but larger in $^{10}\text{Be}-^{12}\text{Be}$ than in ^{12}C , in qualitative agreement with shell model calculations.

To get a good two-neutron separation energy in ^{14}Be and to get a consistent description of ^{12}Be when it is considered as the core of ^{14}Be or described as $^{10}\text{Be} + \text{two neutrons}$, one has to assume an inversion of $2s-1p_{1/2}$ shells in $^{12}\text{Be}-^{13}\text{Be}$ as it is in ^{10}Li and ^{11}Be . This inversion is also suggested by a recent measurement [33] of the transition $2^+(2.1 \text{ MeV}) \rightarrow 0^+(\text{gs})$ in ^{12}Be . The $B(E2)$ is found to be the same as for the transition $2^+(3.3 \text{ MeV}) \rightarrow 0^+(\text{gs})$ in ^{10}Be , suggesting the same effect of particle-phonon couplings in the two systems, $^{11}\text{Be}-^{13}\text{Be}$, therefore the same shells inversion. Furthermore strong particle-particle RPA correlations are known to modify the one-neutron mass operator [38,39] therefore to give further corrections to the one-neutron single energies. They may enhance the inversion process studied in refs.[11,34] for ^{11}Be , even though when adding the two contributions due to couplings with phonons and pair vibrations one has to subtract the second order term in order to eliminate double counting, so that only very mixed RPA states will contribute. From the calculated amplitudes one may expect this contribution to be non negligible in ^{11}Be and ^{13}Be and even larger in ^{13}Be than in ^{11}Be .

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